Spatial Filtering Methods in MEG
Part 1: Theory and Background

Douglas Cheyne, PhD
Program in Neurosciences and Mental Health
Hospital for Sick Children Research Institute
&
Department of Medical Imaging
University of Toronto
MagnetoEncephaloGraphy (MEG)

Neuronal currents generate magnetic fields measurable outside of the head

Temporal waveforms

Spatial patterns

latency = 40 ms

Source Reconstruction
Source Imaging using MEG

Inverse Problem (Helmholtz, 1853)

To determine the underlying current sources in head from the measured external potentials or fields

• Solutions are non-unique
• May be highly underdetermined

Solution:
Construct “forward” models of the sources and fit models to data
Part 1: The forward model

What are we modeling in MEG?
Neural generation of the MEG signal

Neuron

Dendritic currents (EPSPs and IPSPs)
- Post-synaptic potentials (EPSPs and IPSPs) generate intracellular (impressed) currents and extracellular (return or volume) currents
- Both can generate magnetic fields outside the head

Axonal currents (action potentials)
- Action potentials produce only weak magnetic fields due to cancellation and short duration
- They do not contribute to the MEG signal
Neural generation of the MEG signal

What generates neuromagnetic fields?

- dendritic currents due to PSPs in pyramidal cells
- organized in functional columns ($\approx 100,000$ neurons / mm$^2$)
- summation of millions of PSPs can generate current densities in range of 0.05 to 0.5 nA-m* / mm$^2$
- typical MEG source strength ranges from 5 – 30 nAm

How large are MEG fields?

Evoked responses

- typical amplitudes of 100 – 300 femtoTesla
- smallest measurable evoked response about 5 fT

Epileptic spikes

- $\approx 1000$ to 2000 femtoTesla
- visible in background MEG
- generated by activation of large cortical areas

* moment = current x length (Ampere-meters)
The forward model

We require an accurate model of the magnetic field at the sensors (sometimes referred to as the “lead field”)

How is this calculated?
Modeling the MEG Signal

We want to know $B(r)$ -- the magnetic field generated by the total current (impressed + volume currents) flowing in a bounded medium (the head).

$J^p$ = primary (impressed) current

$J^v$ = volume (passive) current

Total current $J^T = J^p + J^v$
Modeling the MEG Signal

The magnetic field \( \mathbf{B}(\mathbf{r}) \) is related to the total current distribution \( J(\mathbf{r}') \) over the volume \( G \) by the integral form of the Biot-Savart Law

\[
\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_{G} \frac{J(\mathbf{r}') \times (\mathbf{r} - \mathbf{r}'))}{|\mathbf{r} - \mathbf{r}'|^3} dV
\]  

(1)

We divide the total current into two parts: the impressed (primary) currents \( J^p \) and passive currents flowing through the volume \( J^v \) where,

\[
J^T = J^p + J^v
\]

\( \mu_0 = \) magnetic permeability of free space \((4\pi \times 10^{-7} \text{Tm/A})\)

Biot-Savart Law (current in a wire)

\[
d\mathbf{B}(\mathbf{r}) = \frac{\mu_0 I d\mathbf{L} \times \hat{r}}{4\pi |\mathbf{r}|^2}
\]
The field due to the primary current \( J^p \) can be calculated separately:

\[
B_0(r) = \frac{\mu_0}{4\pi} \int_G \frac{J^p(r') \times (r - r')}{|r - r'|^3} dv
\]  

Further, if the impressed current represents a focal source we can replace the volume integral of primary current \( J^p(r') \) with a point source represented by the dipole \( q = I(r_{\text{source}} - r_{\text{sink}}) \)

\[
B_0(r) = \frac{\mu_0}{4\pi} \frac{q \times (r - r')}{|r - r'|^3}
\]

In an unbounded medium this would describe the total magnetic field!
However in a bounded medium there are passive volume currents. How to deal with these?

From the “quasi-static” case of Maxwell’s equations it can be shown that volume currents are related to the electric field which is proportional to the gradient of the electric potential:

\[ J^v(r') = -\sigma \nabla V(r') \]

where \( \sigma \) is the conductivity of the medium.

Substituting into Eq. (1)

\[
B(r) = \frac{\mu_0}{4\pi} \int_G \left[ J^p(r') - \sigma \nabla V(r') \right] \times \left( \frac{(r-r')}{|r-r'|^3} \right) dv
\]

We can separate out terms for the primary and volume currents:

\[
B(r) = B_o(r) - \frac{\mu_0}{4\pi \sigma} \int_G \frac{\nabla V(r') \times (r-r')}{|r-r'|^3} dv
\]  

(4)
Since the gradient of the potential is only non-zero at boundaries, we can replace the volume integral with surface integrals. These are termed “fictitious” or secondary currents because they lie on (and are normal to) the boundary (i.e., are not real physical currents)

\[ B(r) = B_0(r) - \frac{\mu_0}{4\pi} \sigma \int_S \frac{V(r')n(r') \times (r - r')}{|r - r'|^3} dS \]

Thus, the volume current contributions for a complex piecewise conductor (like the head) can be calculated by summing the potentials over all surfaces. This requires using a boundary element method (BEM) applied to a real physical model of the head (e.g., triangulated meshes of the brain, CSF, skull, scalp)

The solution for m surfaces:

\[ B(r) = B_0(r) - \frac{\mu_0}{4\pi} \sum_{j=1}^{m} (\sigma_j^- - \sigma_j^+) \int_{S_j} \frac{V(r')n(r') \times (r - r')}{|r - r'|^3} dS_j \]
Thus, with a realistic model we can compute the external magnetic field due to all currents from Eq. 5. In addition, solution for scalp surface potential provides a forward model for EEG source analysis!

However, that’s pretty complicated and there are some drawbacks:

1) Requires accurate surfaces as well as conductivity estimates for the model
2) Solution accuracy degrades for sources that are close to the boundaries
3) Computationally time-consuming

Some workarounds:

- Warp predefined (canonical) meshes to individual brains
- Use single boundary only (e.g., skull) for MEG
Since the head is approximately spherical in shape, a single sphere conductor model has been used extensively in MEG. This simplifies the forward model, but also has additional implications for what we can measure!

Case 1: For radially oriented current source in a perfect sphere:

Due to symmetry, no external magnetic field is detected as the primary and volume currents cancel each other.

(Note: Since both primary and secondary (fictitious) sources are radially oriented we would not detect either with a radially oriented detector in any case, as the field lines are orthogonal to axis of the detector.)
The Spherical Model:

Since the head is approximately spherical in shape, a single sphere conductor model has been used extensively in MEG.

This simplifies the forward model, but also has additional implications for what we can measure!

Case 2: For tangentially oriented current source in a perfect sphere:

A) An external magnetic field is detected due to both primary and volume currents

B) A radially oriented detector is only sensitive to the primary current, as all volume (secondary) currents are radial. Thus, only a tangentially oriented detector measures volume currents!
The Spherical Model:

Most importantly, the calculation of both primary and volume contributions to the magnetic field are simplified and do not require knowing the conductivity of the medium!

The solution for a dipole source in a single conducting sphere is given by Sarvas (1987):

\[
B(r) = \frac{\mu_0}{4\pi F^2} \left\{ F q \times r_o - \left[ (q \times r_o) \cdot r \right] \nabla F \right\}
\]

\[
F = a \left( r a + r^2 - r_o \cdot r \right) \]

\[
\nabla F = \left( r^{-1} a^2 + a^{-1} a \cdot r + 2a + 2r \right) r - \left( a + 2r + a^{-1} a \cdot r \right) r_o \]

- \( q = \text{dipole moment (A-m)} \)
- \( \mu_0 = \text{permeability of free space} = 4\pi \times 10^{-7} \text{ Tm/A} \)
- \( a = |a| \)
- \( r = |r| \)
The “Multisphere” head model

- attempts compromise between realistic and single-sphere model
- provides small (?) correction for non-spherical portions of head
- does not provide globally optimum solution for all sources

* Note: definition of purely ‘radial’ or ‘tangential’ source is now ambiguous:
  Solution: Use mean sphere center to define dipole orientation
Other Corrections to Forward Model

1. The primary detectors are often configured as gradiometers (e.g., CTF uses 1st order gradiometers) rather than magnetometers. This requires computing field at both coils and taking difference.
Magnetometers vs. Gradiometers

Magnetometer
- measures field $B$

1st-order gradiometer
- measures gradient $\frac{dB}{dx}$

Distance from source (m)
Sickkids 151 channel CTF System – Sensor geometry
Other Corrections to Forward Model

1. The primary detectors are often configured as gradiometers (e.g., CTF uses 1st order gradiometers) rather than magnetometers. This requires computing field at both coils and taking difference.

2. If noise subtraction involving reference channels is used (“field balancing”, “synthetic gradients”) field at reference channels must be calculated and same transformation applied to the lead field.
Corrections to Forward Solution

Magnetometers located further away from the head can be used to measure and subtract noise from the primary sensors. This is referred to as “field balancing.”

There are two types:

- adaptive balancing (adjust magnetometer weights to empty room noise)
- synthetic gradients (fixed weights that may use higher order references)
Part 2: The inverse solution

Main approaches to source modeling in MEG:

Dipole fitting (parametric) methods
- simple method, well understood
- need to know number of independent sources
- difficult to combine results across subjects

Linear estimation (minimum-norm)
- no need to specify number of sources
- can model distributed sources
- requires complex pseudoinverse (regularization)
- generally requires artifact-free data

Spatial filtering (beamforming)
- adaptive ("data-driven") – stable solutions
- suppresses artifacts in data automatically
- performance degrades for highly synchronous sources
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Source Modeling using Adaptive Spatial Filtering (Beamforming)

What is a beamformer and how does it work?
Spatial filters can be conceptualized as a type of projection operator in multidimensional “signal space”.
Dipole search vs Spatial Filter in Signal Space

Dipole Search
• select number of dipole sources
• manipulate dipole parameters to match data
  – requires *a priori* selection of # of sources
  – limited to small number of sources
  – requires high SNR and artifact free data

Spatial Filter
• match data to forward solution for fixed source
• output of filter over time acts as “virtual sensor”

Volumetric imaging with a spatial filter
• create lattice of spatial filters throughout brain
• generate source image from output
Signal Space

Signal Space projections can describe differences between patterns of signals (e.g., forward models) and introduce some useful concepts:

Signal Space Angle:
The angle between two signal space vectors. Can be used to determine how well separated (uncorrelated) sources are independent of amplitude.

Orthogonal Projection:
If two signal space vectors are orthogonal (angle = 90°) their sources are independent (change in one does not induce a change in the other)

Signal subspace:
A reduced dimension space (e.g., a plane in 3 dimensional space). Projecting signal onto a subspace gives component explained entirely by the linear combination of basis vectors that make up the subspace
A spatial filter is the weighted output of the MEG sensor array that reflects activity at a specific brain location over time.

**Signal Space Projection (SSP)**

\[ W(r) = L(r) = \text{lead field of source} \]

**Beamformer**

\[ W(r) = L(r) \frac{C_m^{-1}}{L(r) C_m^{-1} L(r)} \]
Spatial filtering methods

Beamformer algorithms differ in terms of forward model used

**Vector beamformers**

*orthogonal current sources at each voxel*

Linearly Constrained Minimum-Variance (LCMV) beamformer
(Van Veen et al., 1997)

**Scalar beamformers**

*compute optimal current direction at each voxel*

Synthetic Aperture Magnetometry (SAM)
(Robinson & Vrba, 1999)

*for EEG # directions = 3, for MEG spherical model, # directions = 2
Introduction of Beamformers in EEG/MEG

Linearly Constrained Minimum-Variance (LCMV) beamformer (Van Veen et al., 1997)
• first application of beamformer method to EEG/MEG inverse problem, EEG only

Synthetic Aperture Magnetometry (SAM) (Robinson & Vrba, 1999)
• introduction of scalar beamformer and use of unaveraged data, “differential” imaging

Dynamic Imaging of Coherent Sources (DICS) (Gross et al., 2001)
• frequency domain beamformer, coherence analysis

Eigenspace / spatiotemporal beamformer (Sekihara et al., 2001, 2002; Dalal et al., 2004)
• derived time-courses for eigenspace beamformer (averaged data), NutMEG toolbox

Event-related SAM (erSAM) (Cheyne et al., 2004, 2006)
• combined SAM algorithm and spatiotemporal approach to image evoked responses

SAMerf (Robinson, 2004)
• SAM algorithm modified for short time windows (not same as erSAM!)

Event-related beamformer (ERB) (Cheyne et al., 2008)
• revised erSAM for optimized orientation calculation

5-D beamformer (Dalal et al., 2010)
• computes beamformer over separate frequency bands and time windows
Spatial filtering methods

Why are there no surface based beamformers?

Cortically constrained beamformers

Problem:

• Deviation from correct orientation significantly attenuates output of beamformer (Hillebrand and Barnes, 2003)

• Requires realistic surface, accurate co-registration between MEG and MRI coordinate systems

• May still be feasible and is under development!

Beamformer solution on cortical surface extracted with CIVET
Spatial filtering methods

How do beamformers work?
Spatial filtering methods

Beamformer can be conceptualized as SSP with orthogonal projection of noise:
Spatial filtering methods

Signal Space Projection

Spatial pattern (e.g., forward solution for source $\theta$) given by $B_\theta$ is used to weight sensor array i.e., project data onto the source vector in "signal space"

$$W_\theta = B_\theta \quad S_\theta(t) = B_\theta^T m(t)$$

Orthogonal Signal Space Projection

Can be used to remove contribution of a known spatial pattern $b_\theta$ (measured or modeled) from the data by projecting data onto subspace orthogonal to this direction in signal space

$$m_{proj}(t) = m(t) - (m(t) \cdot b_\theta^T) b_\theta^T$$
Spatial filtering methods

Signal Space Projection

- output over time is correlation (projection) between data and forward solution

- assumes no other sources sources are uncorrelated or orthogonal in signal space

- non-orthogonal (ie., spatially correlated) source will distort output of SSP
Spatial filtering methods

Null-steering beamformer

- projects data into subspace orthogonal to all “noise” sources thereby “nulling” out contribution of interference source
- then project residual onto target source
Spatial filtering methods

How do we obtain optimal weights $W(r)$ when number and location of interfering sources are unknown?
Adaptive (Minimum-variance) Beamforming

Calculation of Spatial Filter:

For location \( r \), define spatial filter as weight matrix, \( \mathbf{W}(r) \)

Filter output as function of time is measured data vector \( \mathbf{m}(t) \) scaled by weights

\[
\mathbf{S}(r, t) = \mathbf{W}(r)^T \mathbf{m}(t)
\]

Dimensions of \( \mathbf{W}(r) = N \) source orientations \( \times \) \( M \) channels

For scalar beamformer (source has single optimized orientation)

\[
\mathbf{s}(r, t) = \mathbf{w}(r)^T \mathbf{m}(t)
\]  

("virtual sensor")  \rightarrow  

\[\text{signal}^\downarrow\]
Total source power emanating from location $r$ over time period $T$ is given by

$$P = \int_{T} \left\| w(r)^T m(t) \right\|^2 dt$$

In matrix notation

$$P = w(r)^T C w(r)$$

where $C = M \text{ channel x M channel covariance matrix}$

For multi-dimensional weights, signal power is given by

$$P = tr \left\{ W(r)^T C W(r) \right\}$$
Choose weights that will minimize total source power (weights x combined activity of all sources = 0)

\[
\min P = \{W(r)\} \mathbf{W}(r)^T \mathbf{C} \mathbf{W}(r)
\]

subject to constraint that they don’t suppress the source of interest (weights x forward model for source at location \( r = 1 \)):

\[
\mathbf{W}(r)^T \mathbf{L}(r) = \mathbf{I}
\]

where \( \mathbf{L}(r) \) = lead field for current dipole(s) at location \( r \)

Solution is given by,

\[
\mathbf{W}(r) = \mathbf{C}^{-1} \mathbf{L}(r) \left[ \mathbf{L}(r)^T \mathbf{C}^{-1} \mathbf{L}(r) \right]^{-1}
\]
Spatial filtering methods – beamforming vs. SSP

Simulation (2 sources)

SSP

Beamformer

Sensitivity of target voxel weights at all other voxels

\[ P = W(r_{\text{target}}) \times L(r_i) \]
Suggested Readings

**Forward modeling:**


**Beamforming Methods:**


